

Calculators and mobile phones are not allowed.

Answer all of the following questions.

1. [6 points each] Evaluate the following integrals:

(a)
$$\int \frac{2x^2 + 3}{x^3 + x^2 + 4x + 4} dx$$

(b)
$$\int \frac{x}{\sqrt{x^2 - 4x + 5}} dx$$

(c)
$$\int \cos 2x \cos 4x dx$$

(d)
$$\int x^3 \tan^{-1}(x^2) dx$$

(e)
$$\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$$

2. [6 points] Determine whether the following integral converges, if it converges, find its value.

$$\int_{-\infty}^{\infty} \sin^2 x dx$$

3. [2+5 points] Let C be the curve given by the parametric equations

$$x = 2 \tan t, \quad y = \tan^2 t; \quad 0 \leq t \leq \pi/3.$$

(a) Find the slope of the tangent line at the point on the curve C that corresponds to $t = \pi/4$.

(b) Find the length of the curve C .

4. [4+3 points] Let R be the region outside the curve $r = 1$ and inside the curve $r = 2 \sin 3\theta$.

(a) Sketch the region R .

(b) Find the area of the region R .